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Solar Pressure Effects for a Constellation in Highly Elliptical Orbit

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I. Introduction

IN HIGHLY elliptical orbits (HEO), the solar pressure can perturb the motion of a constellation because of the longer exposure of the satellites to the sun [1]. To study its effects on the controller, the Tschauner–Hempel (TH) equations are augmented to include the effects of the solar pressure force. These TH equations are also expressed to include the effects of the perturbation due to the oblateness of the Earth (or J2 perturbation) [2]. To formulate this problem, the Carter–Humi (CH) approach [3] is used in which the TH equations are expressed in terms of the eccentricity and the true anomaly angle. These terms are important for the development of the control scheme [4–6].

The TH equations contain nonlinear terms due to the J2 perturbation [2] that can be considered in the controller. Capó-Lugo and Bainum [2] developed a hierarchical control scheme, in discrete format, to maintain the separation distance constraints for the nonlinear TH equations. This controller is based on the linear quadratic regulator and compensates for these nonlinear effects. The solar pressure force is only dependent on the position of the satellite but can affect the control effort. In conclusion, this paper shows how the solar pressure effects can cause variations in the control effort for the correction of the separation distance constraints for a pair of satellites within a constellation.

II. Tschauner–Hempel Equations for a Perturbed Motion

Following the Carter–Humi approach [3], the TH equations for a perturbed motion, which defines the motion of a pair of satellites about the Earth when the J2 perturbation [2] and the solar pressure force are present, can be written as

$$\begin{aligned} y_1'' &= 2y_2' + \frac{3B_2k_h^2}{2h\kappa}y_1(1\text{–}5\sin^2 i) + a_1 + c_1 \\ y_2'' &= -2y_1' + 3\kappa y_2 - \frac{15B_2k_h^4}{2\mu\kappa}y_3^2 - \frac{3B_2k_h^2}{h\kappa}y_2(2\text{–}15\sin^2 i) \\ &\quad - \frac{15B_2}{h^2\kappa}\sin^2 i + a_2 + c_2 \\ y_3'' &= -y_3 - \frac{3B_2k_h^2}{2h\kappa}y_3(1\text{–}5\sin^2 i) + \frac{15B_2k_h^2}{h\kappa}y_2\sin i + a_3 + c_3 \end{aligned} \tag{1}$$

where

$$\begin{aligned} a_j &= \left(\frac{h^6}{\mu^4}\right)\frac{T_m}{(1+e\cos f)^3 m_0}u_j(f) = bu_j(f)\kappa^3 \\ c_j &= \left(\frac{h^6}{\mu^4}\right)\frac{F_{Sj}}{m_0(1+e\cos f)^3} \qquad j = 1,2,3 \end{aligned} \tag{2a}$$

$$k_h = \frac{\mu}{h^{3/2}} \qquad \kappa = \frac{1}{1+e\cos f} \qquad b = \left(\frac{h^6}{\mu^4}\right)\frac{T_m}{m_0} \qquad B_2 = -\frac{2}{3}J_2R_E \tag{2b}$$

$$F_{Sj} = f_{SR,j} - f_{SM,j} \qquad u_j(f) = \frac{T_j(f)}{T_m} \tag{2c}$$

$$(\,)' = \frac{d(\,)}{df} \qquad y_j = (1+e\cos f)x_j \tag{2d}$$

κ is determined from the equation of a Keplerian orbit where $\mu = GM_E$, r is the radial position of the satellite at any point in the orbit, R_E is the Earth radii, and h is the angular momentum. x_1 is positive against the motion of the spacecraft, x_2 is positive along the radial direction, and x_3 is positive when the right-handed system is completed. The TH equations are expressed in the x_1, x_2, x_3 system which contains the actual separation distance between the maneuvering and the reference spacecraft and is defined in the time domain. By transforming the TH equations from the time into the true anomaly angle domain and by using the y_1, y_2, y_3 equations [Eq. (2d)] as shown in [3], the TH equations can be reduced into a compact system of equations [Eq. (1)]. The maneuvering spacecraft is assumed to have an applied thrust vector along the reference coordinate system, and the reference spacecraft is initially assumed to be acted on by a Newtonian gravitational force directed toward the center of the Earth. It is assumed that the actual mass ($m(f)$) of the satellite can be approximated to its initial mass ($m(f) \approx m_0$), and T_m is the maximum thrust that can be applied by the thruster. F_{Sj} (or $\mathbf{F}_S$) is the solar pressure force in which $F_{SR,j}$ (or $\mathbf{F}_{SR}$) and $F_{SM,j}$ (or $\mathbf{F}_{SM}$), respectively, is the solar pressure due to the reference and the maneuvering spacecraft.

III. Solar Pressure in Formation Flying

The solar pressure force is due to the incidence of light on the surface of a satellite. This incidence of light creates outside forces or

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torques that can change the orbital and attitude dynamics of the satellite. Karymov [7] developed a formulation to define the forces and torques due to this perturbation. This formulation depends on the shape of the satellite, but the direction of the solar rays is defined in terms of the position of the satellite. The force due to the incidence of light over the surface of an assumed opaque satellite can be expressed as

$$\mathbf{F}^+ = -h_0\hat{\sigma}\int_S (\hat{\sigma}\cdot\hat{n})\,\mathrm{d}S \tag{3a}$$

$$\mathbf{F}^- = -2h_0\int_S \hat{n}(\hat{\sigma}\cdot\hat{n})^2\,\mathrm{d}S \tag{3b}$$

where

$$h_0 = \frac{E_0}{\upsilon_0}\left(\frac{r_0}{R_S}\right) \approx \frac{E_0}{\upsilon_0} = 4.72\times 10^{-7}\ \frac{\mathrm{kg}}{\mathrm{m}^2} \tag{4}$$

$\mathbf{F}^+$ is the force for a total absorbing surface in which the reflectivity coefficient (ε) is equal to zero. $\mathbf{F}^-$ is the force for a total reflective surface such that $\varepsilon = 1$. $\hat{\sigma}$ is the unit vector representing the direction of the incident light over the body of the satellite and is represented as $\hat{\sigma} = a_0\hat{i} + b_0\hat{j} + c_0\hat{k}$. The values a_0, b_0, and c_0 represent the direction cosines in the X, Y, and Z directions of the incident light, respectively; in addition, $a_0^2 + b_0^2 + c_0^2 = 1$. $\hat{n}$ is the unit vector normal to the surface of the body of the satellite. E_0 is the magnitude of the flux of light energy arriving at a unit surface of the body, R_S is the distance of the body to the sun, r_0 is the distance from the Earth to the sun, and υ_0 is the velocity of light in a vacuum. In Eq. (4), it is assumed that the relative distance from the satellite to the sun and the relative distance from the Earth to the sun will not vary greatly [8]. The general force [7] due to the solar pressure for an arbitrary surface can be expressed as

$$\mathbf{F}_S = (1-\varepsilon)\mathbf{F}^+ + \varepsilon\mathbf{F}^- \tag{5}$$

Equation (5) will be used to represent the forces due to the solar pressure for both satellites.

A model of the spacecraft must be used here to determine the forces due to the solar pressure with Karmyov's theory [7] [Eqs. (3)]. For this work, the proposed Aurora Multiscale Mission [9] (AMM) is chosen. In the AMM, the spacecraft has the form of a hexagon, but it is assumed as a right circular cylinder. The diameter and height of the cylinder, respectively, are 40 in. (1.016 m) and 22 in. (0.5588 m). The spacecraft have a mass of 90 kg each.

To determine the solar pressure force with Eqs. (3), the surface integral must be divided in two different integrals containing the lateral surface of the body of the cylinder and the top (or bottom) of the cylinder. The surface integral in the Cartesian (x, y, z) coordinate system is complicated, but this surface integral can be transformed to cylindrical coordinates to perform this calculation. This coordinate system is located at the center of gravity of the cylinder. The equations defining the right circular cylinder can be represented as

$$x^2 + y^2 = r_c^2, \qquad z = \frac{l}{2} \tag{6}$$

Solving Eqs. (3), the forces due to a total absorbing and reflective surface for a single satellite are

$$\mathbf{F}^+ = \mathbf{F}_B^+ + \mathbf{F}_T^+ \qquad \mathbf{F}^- = \mathbf{F}_B^- + \mathbf{F}_T^- \tag{7a}$$

where

$$\begin{aligned}\mathbf{F}_B^+ &= -4(lR_c)b_0h_0\hat{\sigma}\\ F_B^- &= -\tfrac{4}{3}(lR_c)h_0[4a_0b_0\hat{i} - (2a_0^2 + 8b_0^3)\hat{j}]\end{aligned} \tag{7b}$$

$$\mathbf{F}_T^+ = -\pi h_0c_0R_c^2\hat{\sigma} \qquad \mathbf{F}_T^- = -2c_0^2h_0(\pi R_c^2)\hat{k} \tag{7c}$$

$\mathbf{F}_B$ and $\mathbf{F}_T$ are the solar pressure forces due to the body and top of the right circular cylinder, respectively.

The direction of the incidence of light must be determined before the difference in the solar pressure forces between the satellites are determined in the TH equations for a perturbed motion. The direction of the incidence of light can vary depending on the position of the satellites along the HEO and on the inclination angle of the sun with respect to the Earth. In [10], it is assumed that the direction of light can be determined with an axis transformation defining the position of the satellites and the inclination angle of the sun with respect to the Earth. Using this assumption, the direction of the light for the reference and target spacecraft can be written, respectively, as

$$a_0 = \cos f\cos i_S \qquad b_0 = \sin f\cos i_S \qquad c_0 = \sin i_S \tag{8a}$$

$$\begin{aligned}a_{0,M} &= \cos(f+\delta f)\cos i_S = (\cos f\cos\delta f - \sin f\sin\delta f)\cos i_S\\ b_{0,M} &= \sin(f+\delta f)\cos i_S = (\sin f\cos\delta f + \cos f\sin\delta f)\cos i_S\\ c_{0,M} &= \sin i_S\end{aligned} \tag{8b}$$

where i_S is the inclination angle of the sun with respect to the Earth, and δf is the difference in the true anomaly angle between the reference and the target satellite. Using the most general triangle that connects the center of the Earth, the maneuvering satellite, and the reference satellite at the vertices of the triangle and assuming that the separation distance between the maneuvering and reference satellite is much less than the distance from the center of the Earth to the reference spacecraft, it can be found that $\delta f \approx 0$. For this reason, the direction of light for the reference spacecraft ($\hat{\sigma}$) is approximately equal to the direction of light of the maneuvering spacecraft ($\hat{\sigma}_M$). Because of these approximations, Eq. (8a) is used to express the direction of light for both satellites. Substituting Eqs. (7) and (8a) into Eq. (2c), the general solar pressure force can be expressed as

$$\mathbf{F}_S = \Delta\varepsilon(\mathbf{F}^- - \mathbf{F}^+) \tag{9a}$$

where

$$\mathbf{F}^+ = -R_ch_0(4lb_0 + \pi R_cc_0)\hat{\sigma} \tag{9b}$$

$$\mathbf{F}^- = R_ch_0\left(-\left(\frac{16}{3}\right)la_0b_0\hat{i} + \left(\frac{4l}{3}\right)(2a_0^2 + 8b_0^3)\hat{j} - \pi R_cc_0^2\hat{k}\right) \tag{9c}$$

and

$$\Delta\varepsilon = \varepsilon_R - \varepsilon_M \tag{9d}$$

ε_R and ε_M are the reflective coefficients for the reference and maneuvering satellite, respectively. To account for differences in the reflectivity coefficients between the maneuvering and reference spacecraft, Eqs. (9) are used to express a variation in the construction of the outside panels.

IV. Hierarchical Control Scheme

The control scheme is expressed in the discrete domain [11] of the true anomaly angle, because the objective of a digital control system is to simplify the implementation in the computer onboard the satellite. A simple algorithm can be developed to perform the control scheme such that the computational time can be reduced. This work uses a discrete form of the TH equations and the cost function [12] to solve the control problem. Using Euler's theorem [12], the discrete form of Eqs. (1) can be expressed as

$$y(k+1) = \lfloor\hat{A}(k)\rfloor y(k) + [\hat{B}(k)]u(k) + \Delta f\psi[y(k)] + \Delta f\Gamma(f(k)) \tag{10a}$$

where

$$[\hat{A}(k)] = I + \Delta f A = \begin{bmatrix} 1 & 0 & 0 & \Delta f & 0 & 0 \\ 0 & 1 & 0 & 0 & \Delta f & 0 \\ 0 & 0 & 1 & 0 & 0 & \Delta f \\ 0 & 0 & 0 & 1 & 2\Delta f & 0 \\ 0 & 3\kappa\Delta f & 0 & 2\Delta f & 1 & 0 \\ 0 & 0 & -\Delta f & 0 & 0 & 1 \end{bmatrix} \tag{10b}$$

$$[\hat{B}(k)] = \Delta f B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ b\kappa^3\Delta f & 0 & 0 \\ 0 & b\kappa^3\Delta f & 0 \\ 0 & 0 & b\kappa^3\Delta f \end{bmatrix} \tag{10c}$$

$$\psi[y(f(k))] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{3B_2k_h^2}{2h\kappa} y_1(1-5\sin^2 i) \\ -\frac{15B_2k_h^4}{2\mu\kappa} y_3^2 - \frac{3B_2k_h^2}{h\kappa} y_2(2-15\sin^2 i) - \frac{15\mu B_2k_h^4}{h^2\kappa}\sin^2 i - \frac{15B_2}{h^2\kappa}\sin^2 i \\ \frac{3B_2k_h^2}{2h\kappa} y_3(3-5\sin^2 i) - \frac{15B_2k_h^4}{h\kappa} y_2 \sin i \end{bmatrix} \tag{10d}$$

$$\Gamma(f(k)) = \left(\frac{h^6}{\mu^4}\right) \frac{\Delta\varepsilon}{m_0(1+e\cos f)^3} \left\{ R_c h_0 \begin{bmatrix} 0 \\ 0 \\ 0 \\ -(\frac{16}{3}) l a_0 b_0 \\ (\frac{4l}{3})(2a_0^2 + 8b_0^3) \\ -\pi R_c c_0^2 \end{bmatrix} + R_c h_0 (4lb_0 + \pi R_c c_0) \begin{bmatrix} 0 \\ 0 \\ 0 \\ a_0 \\ b_0 \\ c_0 \end{bmatrix} \right\} \tag{10e}$$

$$y(k) = [\, y_1(k) \quad y_2(k) \quad y_3(k) \quad y_1'(k) \quad y_2'(k) \quad y_3'(k) \,]^T \tag{10f}$$

I is a 6×6 identity matrix.

The discrete cost function [12] in terms of the CH control approach [3] can be written as

$$J(k) = \frac{\Delta f}{2} \sum_{k=0}^{N_f - 1} (y(k) - y_D)^T \tilde{Q}(k)(y(k) - y_D) + (u(k))^T \tilde{R}(k)(u(k)) \tag{11}$$

where

$$\tilde{Q}(k) = Q\kappa(k) \qquad \tilde{R}(k) = R\kappa^2(k)$$

$$\kappa = \frac{1}{1 + e\cos f(k)} \qquad f(k) = f_L + k\Delta f$$

$y(k)$ and y_D are, respectively, the state and desired state vectors with dimensions $n \times 1$. Q and R are the weights due to the states and the control function, respectively. Q has dimensions $n \times n$, and R has dimensions $n \times 1$. f_L is the initial true anomaly angle, and Δf is the sampling interval in the true anomaly angle. k is an integer value representing the sample which is obtained at every sampling interval in the true anomaly angle. k ranges from 0 to $N_f - 1$, where N_f is the last sample obtained at the final true anomaly angle in the solution of the control problem and is expressed as

$$N_f - 1 = \frac{f_F - f_L}{\Delta f}$$

where f_F is the final true anomaly angle.

The existence of the control problem is obtained from the solution of the discrete Hamilton–Jacobi equation which is defined everywhere to obtain the minimum conditions. The discrete Hamilton–Jacobi equation [13] can be written as

$$H(k) = \frac{\Delta f}{2}[(y(k) - y_D)^T \tilde{Q}(y(k) - y_D) + u^T(k)\tilde{R}u(k)] + p^T(k+1)[\hat{A}(k)y(k) + \hat{B}(k)u(k)] + p^T(k+1)[\Delta f \psi[\bar{y}(k)]] + \gamma(k+1)[y(k) - \bar{y}(k)] \tag{12}$$

where $p(k+1)$ is the costate variable, and $\gamma(k+1)$ is the error vector. In Eq. (12), $\bar{y}(k)$ is the same as $y(k)$, but, for the hierarchical control scheme, these two vectors are used differently. In the first run, $\bar{y}(k)$ is set to zero to solve the system of equations in the linear range; in this way, the solution of the control problem takes advantage of the linear form of the TH equations. In the following executions, $\bar{y}(k)$ is updated with the known states [or $y(k)$] to correct for the nonlinear terms in the state equation. For this reason, the difference between $y(k)$ and $\bar{y}(k)$ is used in the discrete Hamilton–Jacobi equation to determine if the error between them in the following runs is small. If the errors between $y(k)$ and $\bar{y}(k)$ are less than a tolerance value, the hierarchical control scheme has achieved a suboptimal solution. This error is added into the Hamilton–Jacobi equation by multiplying $\gamma(k+1)$ to this difference in which $\gamma(k+1)$ is a Lagrange multiplier.

The minimum principles are used to obtain the necessary conditions to obtain a suboptimal control problem, and, for the hierarchical control scheme, the solution of the discrete control problem [2] becomes as

$$K(k) = \Delta f \tilde{Q} + A^T(k)K(k+1)[I + W(k)K(k+1)]^{-1}A(k) \tag{13a}$$

$$g(k) = -\Delta f \tilde{Q} y_D + \hat{A}^T(k)g(k+1) + \gamma(k+1) + \hat{A}^T(k)K(k+1)[I + W(k)K(k+1)]^{-1}[-W(k)g(k+1) + \Delta f \psi[\bar{y}(k)] + \Delta f \Gamma(f(k))] \tag{13b}$$

$$y(k+1) = [I + W(k)K(k+1)]^{-1}[\hat{A}(k)y(k) - W(k)g(k+1) + \Delta f \psi[\bar{y}(k)] + \Delta f \Gamma(f(k))] \tag{13c}$$

$$u(k) = -(\Delta f \tilde{R})^{-1} B^T(k)[K(k+1)y(k+1) + g(k+1)] \tag{13d}$$

$$\gamma(k+1) = \Delta f J^T[K(k+1)y(k+1) + g(k+1)] \tag{13e}$$

$$W(k) = B(k)(\Delta f \tilde{R}(k))^{-1} B^T(k) \tag{13f}$$

$K(k)$ and $g(k)$ are the Riccati and adjoint Riccati equations, respectively. J is the Jacobian matrix [2] obtained from $\psi[\bar{y}(k)]$. The recursive method is known as a two-level hierarchical control scheme [13], and Fig. 1 shows how to solve the discrete control

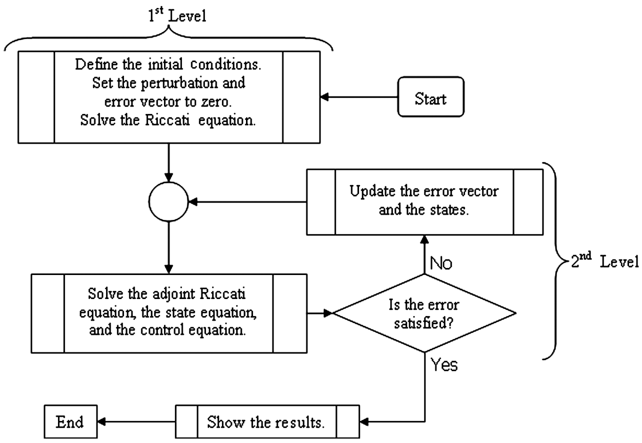

Fig. 1 Flow chart of the two-level hierarchical control scheme.

problem. The main advantage of the hierarchical controller is shown in Eq. (13a). To add the nonlinear effects, Eq. (13b) is continuously updated with the previous states of $\bar{y}(k)$ until the mean square error between the actual and previous state vector has been satisfied as shown in Fig. 1. Capó-Lugo and Bainum in [2] provide more details about the solution of this control problem.

V. Results

The hierarchical control scheme is tested for the drift correction. The drift correction is defined as the difference between the desired and initial coordinates of any pair of satellites within a constellation. The objective of the controller is to correct this drift in minimum time. For the drift correction, y_D is set to zero. The simulation of the hierarchical control scheme is tested with one specific size of the NASA benchmark problem [14] having the following orbital dimensions:

$$a = 42,095.7 \text{ km}; \quad e = 0.818; \quad i = 18.5 \text{ deg}; \quad r_a = 12R_E$$

where a is the semimajor axis, e is the eccentricity, and r_a is the radius of apogee. Thrust levels used for this simulation are comparable to those generated by the ion thrust which is assumed to have a maximum thrust [15] of 0.5 N. The initial mass of the satellite is assumed to be 90 kg. The inclination angle of the sun with respect to the Earth is assumed to be 23.5 deg.

In [4], the drift correction is performed in a minimum time when the varying coefficients for the position of the satellite in the Q matrix are weighted more than the constant coefficients for the velocity of the satellite while the R matrix is maintained with a minimum weight. This weighting for the Q matrix is used because the coefficients in the position are multiplied by the varying term, κ. The Q and R matrices, respectively, are 6×6 and 3×3 diagonal matrices. The weight [16] in the Q matrix is diag$[20 \quad 20 \quad 20 \quad 1 \quad 1 \quad 1]$, and the weight of the R matrix is diag$[1 \quad 1 \quad 1]$. The sampling in the true anomaly angle will be set to 0.05 rad [2]. To implement this control scheme in the computer on board the satellite, it is required to transform from the time domain into the true anomaly angle domain with Kepler's transcendental equation, but this transformation is not pursued in this work because of the complexity to obtain the solution.

The conditions used to initialize the simulations with the hierarchical controller are the following:

$$y(f_L) = [-0.215 \text{ (km)} \quad 0.055 \text{ (km)} \quad 0.002 \text{ (km)} \quad 3.91e-5 \text{ (km/s)} \quad -2.55e-6 \text{ (km/s)} \quad -1.64e-6 \text{ (km/s)}]^T$$

These conditions are obtained from previous simulations with the Satellite Tool Kit software [17] when a pair of satellites first violates the separation distance conditions. If the transformation in Eq. (2d) is used, the dimensions for the state vector $y(f)$ [shown in $y(fL)$] are the same for the state vector $x(f)$.

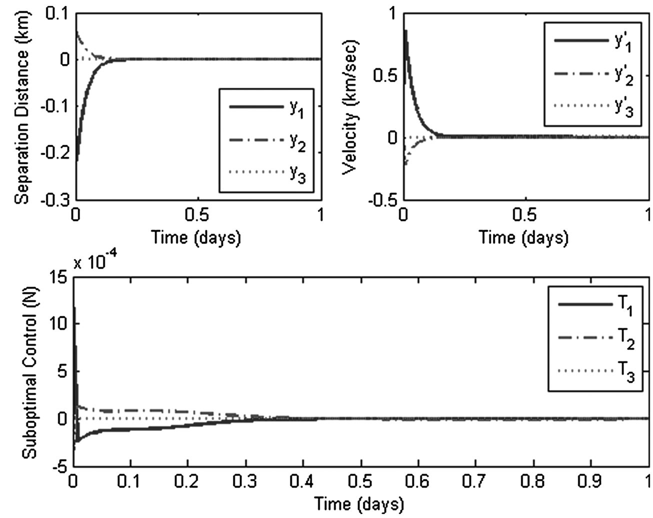

Fig. 2 Drift correction for the station-keeping process when $\Delta\varepsilon = 0.01$.

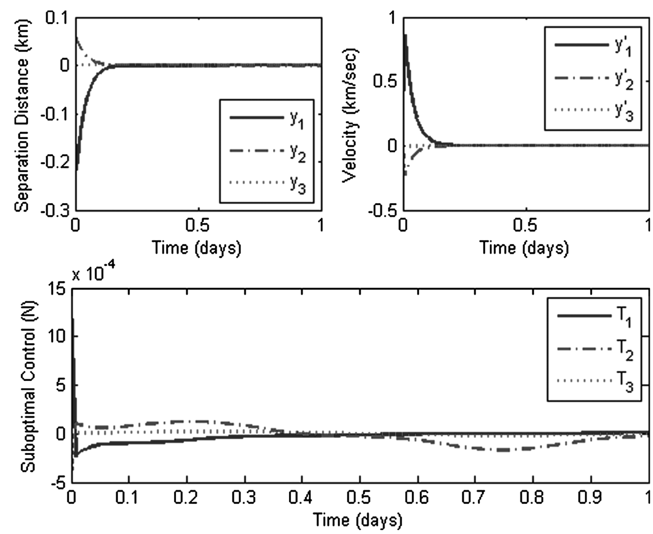

Fig. 3 Drift correction for the station-keeping process when $\Delta\varepsilon = 0.1$.

Figure 2 shows the solution for the drift correction when $\Delta\varepsilon = 0.01$. The drift correction of the separation distances and velocities is performed before the satellites reach the following apogee point. The maximum magnitude of the controller acceleration is 10^{-4} (km/s^2) and shows a stable condition before the pair of satellites reaches the perigee point as shown in Fig. 2. The solar pressure is acting on the satellite along the orbit, and the controller compensates for this perturbation to reduce the drifts for this pair of satellites.

A second simulation is performed using the same initial conditions for the drift correction, but the difference in the reflective coefficient is increased to 0.1. In Fig. 3, the drift correction of the separation distance and velocity is similar as in Fig. 2, but the controller response is different between Figs. 2 and 3. The controller in Fig. 3 mainly compensates for the solar pressure acting along the HEO. Because the difference in the reflectivity coefficient is small in Fig. 2, the solar pressure does not highly affect the pair of satellites along the HEO, and the controller comes into a stable state before the perigee point. It can be observed that, as the difference in the reflectivity coefficient increases, the controller does not show a stable condition because the differential solar pressure has a more noticeable effect on the pair of satellites along the HEO.

VI. Conclusions

The solar pressure force affects the separation distance of a pair of satellites within a constellation in highly elliptical orbits. For this reason, the Tschauner–Hempel equations are augmented to include the effects of the sun. To maintain the separation distance conditions, the hierarchical control scheme is developed to compensate for the nonlinear terms due to the oblateness of the Earth and the solar pressure force. The controller requires a higher effort to correct the solar pressure effects because of the longer exposure to the sun. Hence, this work contributes, for the first time, the development of the Tscahuner–Hempel equations to include the solar pressure effects in formation flying and shows the effects of the solar pressure force on the controller to perform the drift correction in a constellation. In addition, this set of Tschauner–Hempel equations can be extended to include other perturbations due to the Earth and/or the moon.

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References

[1] Capó-Lugo, P. A., and Bainum, P. M., "Implementation of the Strategy for Satisfying Distance Constraints for the NASA Benchmark Tetrahedron Constellations," *Astrodynamics 2005*, edited by B. G. Williams, L. A. D'Amario, K. C. Howell, and F. R. Hoots, Vol. 123, Pt. 2, Advances in the Astronautical Sciences, American Astronautical Society, San Diego, CA, 2005, pp. 1463–1482.

[2] Capó-Lugo, P. A., and Bainum, P. M., "Development and Control Solution for the Perturbed Motion of a Constellation in an Elliptical Orbit," AAS Paper 07-298, 2007.

[3] Carter, T., and Humi, M., "Fuel-Optimal Rendezvous Near a Point in General Keplerian Orbit," *Journal of Guidance, Control, and Dynamics*, Vol. 10, No. 6, Nov.–Dec. 1987, pp. 567–573. doi:10.2514/3.20257

[4] Capó-Lugo, P. A., and Bainum, P. M., "LQR Active Control Schemes to Satisfy the Separation Distance Constraints of the NASA Benchmark Tetrahedron Constellations," *Journal of Guidance, Control, and Dynamics*, Vol. 30, No. 4, July–Aug. 2007, pp. 1152–1156. doi:10.2514/1.24371

[5] Inalhan, G., Tillerson, M., and How, J. P., "Relative Dynamics and Control of Spacecraft Formations in Eccentric Orbits," *Journal of Guidance, Control, and Dynamics*, Vol. 25, No. 1, Jan.–Feb. 2002, pp. 48–59. doi:10.2514/2.4874

[6] Marec, J. P., *Optimal Space Trajectories*, Elsevier, New York, 1979, Chap. 10, pp. 227–262.

[7] Karymov, A. A., "Determination of Forces and Moments due to Light Pressure Acting on a Body in Motion in Cosmic Space," *Prikladnaya Matematika i Mekhanika*, (Journal of Applied Mathematics and Mechanics), Vol. 26, No. 5, 1962, pp. 867–876.

[8] Burns, R., Gabor, M. J., McLaughlin, C. A., Luu, K. K., and Sabol, C., "Solar Radiation Pressure Effects on Formation Flying of Satellites with Different Area to Mass Ratios," AIAA Paper 2000-4132, 2000.

[9] Bainum, P. M., Strong, A., Tan, Z., and Capó-Lugo, P. A., "Techniques for Deploying Elliptically Orbiting Constellations in Along-Track Formation," *Acta Astronautica*, Vol. 57, 2005, pp. 685–697. doi:10.1016/j.actaastro.2005.03.065

[10] Wong, C.-H., and Ahn, H.-S., "Nonlinear Orbital Dynamic Equations and State-Dependent Riccatti Equation Control of Formation Flying Satellites," *Journal of Astronautical Sciences*, Vol. 51, No. 4, Oct.–Dec. 2003, pp. 433–449.

[11] Phillips, C. L., and Nagle, H. T., *Digital Control System Analysis and Design*, 3rd ed., Prentice–Hall, Upper Saddle River, NJ, 1995, Chap. 10, pp. 382–429.

[12] Capó-Lugo, P. A., and Bainum, P. M., "Digital LQR Active Control Schemes to Satisfy the Separation Distance Constraints of the NASA Benchmark Tetrahedron Constellations," AIAA Paper 2006-6014, 2006.

[13] Hassan, M. F., and Singh, M., "The Optimization of Non-Linear System Using a New Two-Level Method," *Automatica*, Vol. 12, 1976, pp. 359–363. doi:10.1016/0005-1098(76)90055-8

[14] Carpenter, R. J., Leitner, J. A., Burns, R. D., and Folta, D. C., "Benchmark Problems for Spacecraft Formation Flying Missions," AIAA Paper 2003-5364, 2003.

[15] Wertz, J. R., and Larson, W. J., *Space Mission Analysis and Design*, 3rd ed., Microcosm Press and Kluwer Academic Publishers, El Segundo, CA, 1999.

[16] Capó-Lugo, P. A., and Bainum, P. M., "Active Control Schemes to Satisfy the Separation Distance Constraints," *Journal of Guidance, Control, and Dynamics*, Vol. 30, No. 4, July–Aug. 2007, pp. 1152–1156. doi:10.2514/1.24371

[17] STK, Satellite Tool Kit Software, Ver. 5.0, Analytical Graphics, Inc., Malvern, PA, 2003.